clumps

12 July 2016

1 DERIVATION

Lets assume that a given clump of radius R_c expands at velocity v adiabatically due to its internal gas pressure:

$$m_p v^2 = 3kT \rightarrow v = \sqrt{\frac{3kT}{m_p}} = \frac{dR}{dt}$$
 (1)

We want $\tau(t)$:

$$\tau(t) = 2\kappa\rho(t)R(t) \tag{2}$$

Lets assume a starting density, ρ_0 so that:

$$\frac{\rho(t)}{\rho_0} = \left(\frac{R_c}{R(t)}\right)^3 \tag{3}$$

$$R(t) = R_c + \sqrt{\frac{3kT}{m_p}} * t \tag{4}$$

so

$$\tau(t) = 2\kappa\rho_0 * R_c^3 \left[R_c + \sqrt{\frac{3kT}{m_p}} * t \right]^{-2}$$
(5)

Based on the simulations of Jiang, Stone and Davis (2014) we can roughly gauge Σ/Σ_0 (Fig 11) and can therefore say that:

$$\rho_0(r) = \rho_{0,s} * \left(\frac{r}{r_s}\right)^{1.75} \tag{6}$$

where the subscript s indicates the value at the spherization radius. Lets now assume that the clump is launched from its position in the disc (with scale height h/r) with the vertical component of the escape velocity:

$$D = r\sqrt{\left(\frac{h}{r}\right)^2 + 1} \tag{7}$$

$$v_{esc} = \sqrt{\frac{2GM}{D}} \tag{8}$$

$$h = v_{esc,y} * t = \frac{1}{\sin\theta} \sqrt{\frac{2GM}{D}} * t \tag{9}$$

the $sin(\theta)$ comes from taking the vertical component of the escape velocity and $\theta = arctan(h/r)$. Note that T is a function of

radius and as we have a large scale height (rad pressure supported flow) goes as $r^{-1/2}$. Using the temperature at the spherization radius ($r_s = \dot{m}_0 * r_{in}$ and \dot{m}_0 is the mass transfer rate in units of Eddington):

$$T = T_s \left(\frac{r}{r_s}\right)^{-1/2} \tag{10}$$

We now need to consider the impulse from the inner regions which has luminosity L_{in} and we assume to radiate isotropically.

$$F = \frac{L_{in}\sigma}{4\pi D^2 c} = \frac{dp}{dt} = \frac{m_p(v_{final} - v_{initial})}{dt}$$
(11)

For simplicity lets assume that the launching happens at the same time as the clump gets removed from the disc so that $v_{initial} \approx 0$. $v_{final} = dz/dt$ where z is in the direction pointing away from the BH radially (i.e. the vertical component is $z * sin(\theta)$):

$$z = \frac{L_{in}\sigma t^2}{8\pi D^2 cm_p} \tag{12}$$

so after a time t for the clump to expand and become optically thin we find its moved vertically to $h + z^* sin(\theta)$ and horizontally to $z^* cos(\theta)$.



Figure 1. clockwise from top-left: accretion at $50 \times Edd$, $100 \times Edd$ and $500 \times Edd$.



Figure 2. clockwise from top-left: accretion at $50 \times Edd$, $100 \times Edd$ and $500 \times Edd$.