## clumps

## 1 DERIVATION

Lets assume that a given clump of radius $\mathbf{R}_{c}$ expands at velocity $v$ adiabatically due to its internal gas pressure:

$$
\begin{equation*}
m_{p} v^{2}=3 k T \rightarrow v=\sqrt{\frac{3 k T}{m_{p}}}=\frac{d R}{d t} \tag{1}
\end{equation*}
$$

We want $\tau(t)$ :

$$
\begin{equation*}
\tau(t)=2 \kappa \rho(t) R(t) \tag{2}
\end{equation*}
$$

Lets assume a starting density, $\rho_{0}$ so that:

$$
\begin{gather*}
\frac{\rho(t)}{\rho_{0}}=\left(\frac{R_{c}}{R(t)}\right)^{3}  \tag{3}\\
R(t)=R_{c}+\sqrt{\frac{3 k T}{m_{p}}} * t \tag{4}
\end{gather*}
$$

so

$$
\begin{equation*}
\tau(t)=2 \kappa \rho_{0} * R_{c}^{3}\left[R_{c}+\sqrt{\frac{3 k T}{m_{p}}} * t\right]^{-2} \tag{5}
\end{equation*}
$$

Based on the simulations of Jiang, Stone and Davis (2014) we can roughy gauge $\Sigma / \Sigma_{0}$ (Fig 11) and can therefore say that:

$$
\begin{equation*}
\rho_{0}(r)=\rho_{0, s} *\left(\frac{r}{r_{s}}\right)^{1.75} \tag{6}
\end{equation*}
$$

where the subscript $s$ indicates the value at the spherization radius. Lets now assume that the clump is launched from its position in the disc (with scale height $\mathrm{h} / \mathrm{r}$ ) with the vertical component of the escape velocity:

$$
\begin{gather*}
D=r \sqrt{\left(\frac{h}{r}\right)^{2}+1}  \tag{7}\\
v_{e s c}=\sqrt{\frac{2 G M}{D}}  \tag{8}\\
h=v_{e s c, y} * t=\frac{1}{\sin \theta} \sqrt{\frac{2 G M}{D}} * t \tag{9}
\end{gather*}
$$

the $\sin (\theta)$ comes from taking the vertical component of the escape velocity and $\theta=\arctan (h / r)$. Note that T is a function of
radius and as we have a large scale height (rad pressure supported flow) goes as $\mathrm{r}^{-1 / 2}$. Using the temperature at the spherization radius $\left(\mathrm{r}_{s}=\dot{m}_{0} * r_{i n}\right.$ and $\dot{m}_{0}$ is the mass transfer rate in units of Eddington):

$$
\begin{equation*}
T=T_{s}\left(\frac{r}{r_{s}}\right)^{-1 / 2} \tag{10}
\end{equation*}
$$

We now need to consider the impulse from the inner regions which has luminosity $L_{i n}$ and we assume to radiate isotropically.

$$
\begin{equation*}
F=\frac{L_{i n} \sigma}{4 \pi D^{2} c}=\frac{d p}{d t}=\frac{m_{p}\left(v_{\text {final }}-v_{\text {initial }}\right)}{d t} \tag{11}
\end{equation*}
$$

For simplicity lets assume that the launching happens at the same time as the clump gets removed from the disc so that $v_{\text {initial }} \approx 0 . v_{\text {final }}=\mathrm{dz} / \mathrm{dt}$ where z is in the direction pointing away from the BH radially (i.e. the vertical component is $z * \sin (\theta))$ :

$$
\begin{equation*}
z=\frac{L_{i n} \sigma t^{2}}{8 \pi D^{2} c m_{p}} \tag{12}
\end{equation*}
$$

so after a time $t$ for the clump to expand and become optically thin we find its moved vertically to $\mathrm{h}+\mathrm{z}^{*} \sin (\theta)$ and horizontally to $\mathrm{z}^{*} \cos (\theta)$.


Figure 1. clockwise from top-left: accretion at $50 \times$ Edd, $100 \times$ Edd and $500 \times$ Edd.


Figure 2. clockwise from top-left: accretion at $50 \times$ Edd, $100 \times$ Edd and $500 \times$ Edd.

